

Derivation of Hydrodynamic Equations for Binary Gas Mixture

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Abstract. Velocities, densities, pressures, stresses, temperatures, heat fluxes and internal energies of each gas are individually defined. Moment equations for mass, momentum and energy of both gases are separately derived on basis of Boltzmann equations. Momentum equations have velocity relaxation terms between different gases and energy equations have velocity and temperature relaxation terms between those.

Keywords: Boltzmann Equations, Gas-Mixture, Hydrodynamic Equations.

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INTRODUCTION

In a book of C. Chapman and T. G. Cowling [1], fluid mechanical equations of a binary gas-mixture are described on the basis of Boltzmann equations. But they assume that temperatures of both gases are equal. In our paper, it is assumed that temperatures are different for both gases. We put distribution functions $f^{(\kappa)}(\mathbf{x}, \mathbf{c}, t)$ ($\kappa = 1, 2$) for each gas of the mixture and each mass velocity defined by

$$\mathbf{v}^{(\kappa)}(\mathbf{x}, t) = \frac{1}{n^{(\kappa)}(\mathbf{x}, t)} \iiint \mathbf{c} f^{(\kappa)}(\mathbf{x}, \mathbf{c}, t) d^3 \mathbf{c} \quad (1)$$

The independent variables \mathbf{x}, \mathbf{c} and t of Boltzmann equations are changed to $\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)} = \mathbf{c} - \mathbf{v}^{(\kappa)}(\mathbf{x}, t)$ and t . Boltzmann equations with the independent variables $\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}$ and t can be written.

$$\left. \begin{aligned} \frac{\partial f^{(\kappa)}}{\partial t} + \left(\mathbf{v}^{(\kappa)} \cdot \frac{\partial}{\partial \mathbf{x}} \right) f^{(\kappa)} - \left(\frac{\partial \mathbf{v}^{(\kappa)}}{\partial t} + \left(\mathbf{v}^{(\kappa)} \cdot \frac{\partial}{\partial \mathbf{x}} \right) \mathbf{v}^{(\kappa)} \right) \cdot \frac{\partial f^{(\kappa)}}{\partial \tilde{\mathbf{c}}^{(\kappa)}} \\ + \tilde{\mathbf{c}}^{(\kappa)} \cdot \frac{\partial f^{(\kappa)}}{\partial \mathbf{x}} - \left(\left(\tilde{\mathbf{c}}^{(\kappa)} \cdot \frac{\partial}{\partial \mathbf{x}} \right) \mathbf{v}^{(\kappa)} \right) \cdot \frac{\partial f^{(\kappa)}}{\partial \tilde{\mathbf{c}}^{(\kappa)}} = \frac{\partial f^{(\kappa)}}{\partial t} + \frac{\partial f^{(\kappa)}}{\partial t} \end{aligned} \right\} \quad (2)$$

where

$$\left. \begin{aligned} \frac{\partial f^{(\kappa)}}{\partial t} = \iiint_{\tilde{\mathbf{c}}^{(\kappa)}} \iiint_{\mathbf{k}=1} \left\{ f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(\kappa')}, t) f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}_2^{(\kappa')}, t) - f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(\kappa)}, t) f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}_2^{(\kappa)}, t) \right\} k_c^{(\kappa \kappa)} d^2 \mathbf{k} d^3 \tilde{\mathbf{c}}_1^{(\kappa)} \\ \tilde{\mathbf{c}}_1^{(\kappa')} = \tilde{\mathbf{c}}_1^{(\kappa)} - \left((\tilde{\mathbf{c}}_1^{(\kappa)} - \tilde{\mathbf{c}}_2^{(\kappa)}) \cdot \mathbf{k} \right) \mathbf{k}, \quad \tilde{\mathbf{c}}_2^{(\kappa')} = \tilde{\mathbf{c}}_2^{(\kappa)} + \left((\tilde{\mathbf{c}}_1^{(\kappa)} - \tilde{\mathbf{c}}_2^{(\kappa)}) \cdot \mathbf{k} \right) \mathbf{k} \end{aligned} \right\} \quad (3)$$

$$\left. \begin{aligned} \frac{\partial f^{(i)}}{\partial t} = \iiint_{\tilde{\mathbf{c}}^{(i)}} \iiint_{\mathbf{k}=1} \left\{ f^{(i)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(i')}, t) f^{(i)}(\mathbf{x}, \tilde{\mathbf{c}}_2^{(i')}, t) - f^{(i)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(i)}, t) f^{(i)}(\mathbf{x}, \tilde{\mathbf{c}}_2^{(i)}, t) \right\} k_c^{(\kappa \kappa)} d^2 \mathbf{k} d^3 \tilde{\mathbf{c}}_1^{(i)} \\ \tilde{\mathbf{c}}_1^{(i')} = \tilde{\mathbf{c}}_1^{(i)} - \frac{2m^{(\kappa)}}{m^{(\kappa)} + m^{(i)}} \left((\tilde{\mathbf{c}}_1^{(i)} - \tilde{\mathbf{c}}_2^{(\kappa)} + \mathbf{v}^{(i)} - \mathbf{v}^{(\kappa)}) \cdot \mathbf{k} \right) \mathbf{k}, \\ \tilde{\mathbf{c}}_2^{(i')} = \tilde{\mathbf{c}}_2^{(\kappa)} + \frac{2m^{(i)}}{m^{(\kappa)} + m^{(i)}} \left((\tilde{\mathbf{c}}_1^{(i)} - \tilde{\mathbf{c}}_2^{(\kappa)} + \mathbf{v}^{(i)} - \mathbf{v}^{(\kappa)}) \cdot \mathbf{k} \right) \mathbf{k} \end{aligned} \right\} \quad (4)$$

and

$$k_C^{(\iota\kappa)} = \left| \mathbf{c}_1^{(\iota)} - \mathbf{c}^{(\kappa)} \right| \frac{b^{(\iota\kappa)} |\partial \mathbf{b}^{(\iota\kappa)} / \partial \chi^{(\iota\kappa)}|}{\sin \chi^{(\iota\kappa)}} \quad (5)$$

where $b^{(ij)}$ and $\chi^{(ij)}$ are respectively the impact parameter and the deflection angle between ι - and κ -molecules and the case for $\iota = \kappa$ is also valid. We have assumed that each component of the gas has each number density, each pressure, each temperature, each stress tensor, each heat flux and each internal energy:

$$\left. \begin{aligned} n^{(\kappa)}(\mathbf{x}, t) &= \iiint f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) d^3 \tilde{\mathbf{c}}^{(\kappa)} \\ p^{(\kappa)}(\mathbf{x}, t) &= \frac{1}{3} m^{(\kappa)} \iiint (\tilde{\mathbf{c}}^{(\kappa)})^2 f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) d^3 \tilde{\mathbf{c}}^{(\kappa)} \\ T^{(\kappa)}(\mathbf{x}, t) &= \frac{m^{(\kappa)}}{3 k_B n^{(\kappa)}(\mathbf{x}, t)} \iiint (\tilde{\mathbf{c}}^{(\kappa)})^2 f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) d^3 \tilde{\mathbf{c}}^{(\kappa)} \\ p_{\lambda\mu}^{(\kappa)}(\mathbf{x}, t) &= -m^{(\kappa)} \iiint \tilde{c}_\lambda^{(\kappa)} \tilde{c}_\mu^{(\kappa)} f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) d^3 \tilde{\mathbf{c}}^{(\kappa)} \\ q_\lambda^{(\kappa)}(\mathbf{x}, t) &= N^{(\kappa)} \iiint \frac{1}{2} m^{(\kappa)} \tilde{c}_\lambda^{(\kappa)} (\tilde{\mathbf{c}}^{(\kappa)})^2 f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) d^3 \tilde{\mathbf{c}}^{(\kappa)} \\ e^{(\kappa)}(\mathbf{x}, t) &= \frac{N^{(\kappa)}}{2 m^{(\kappa)}} k_B T^{(\kappa)}(\mathbf{x}, t) \end{aligned} \right\} \quad (6)$$

where $N^{(\kappa)}$ is the number of total degrees of freedom of κ -molecule with translational and internal motions. One of the authors (S.K.) obtained dynamics of multi-component plasma by use of the above prescription of macroscopic variables from phenomenological point of view[2].

EQUATIONS OF CONTINUITY AND EQUATIONS OF MOTION

We would derive the equations of change of molecular properties of each species, *i.e.* the moment equations of the molecular properties of each species. The molecular properties concerning the κ -gas are the followings:

$$\phi^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) = 1, \quad m^{(\kappa)} \tilde{\mathbf{c}}^{(\kappa)}, \quad \text{and} \quad \frac{1}{2} m^{(\kappa)} (\tilde{\mathbf{c}}^{(\kappa)})^2 \quad (7)$$

which are all summational invariants. For this case, it is noted that

$$\iiint \phi^{(\kappa)} \frac{\partial_C^{(\kappa)} f^{(\kappa)}}{\partial t} k_C^{(\kappa\kappa)} d^2 \mathbf{k} d^3 \tilde{\mathbf{c}}_1^{(\kappa)} = 0, \quad \iiint \phi^{(\kappa)} \frac{\partial_C^{(\iota)} f^{(\kappa)}}{\partial t} k_C^{(\iota\kappa)} d^2 \mathbf{k} d^3 \tilde{\mathbf{c}}_1^{(\iota)} \neq 0 \quad (8)$$

The first one comes from the summational invariants for the self-collision, *i.e.* $\phi^{(\kappa)}$'s do not change before and after the self-collision. The second shows it is not so for the cross integration between the different species. We want to investigate the equation of change of molecular property $\phi^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t)$ of the κ -molecule, which is assumed to summational invariant:

$$\begin{aligned}
& \left. \begin{aligned}
& \frac{\partial \overline{\phi^{(\kappa)}} \mathbf{n}^{(\kappa)}}{\partial t} + \mathbf{v}_\lambda^{(\kappa)} \frac{\partial \overline{\phi^{(\kappa)}} \mathbf{n}^{(\kappa)}}{\partial \mathbf{x}_\lambda} + \frac{\partial}{\partial \mathbf{x}_\lambda} \left(\mathbf{n}^{(\kappa)} \overline{\phi^{(\kappa)}} \tilde{\mathbf{c}}_\lambda^{(\kappa)} \right) - \mathbf{n}^{(\kappa)} \left\{ \frac{\partial \overline{\phi^{(\kappa)}}}{\partial t} + \mathbf{v}_\lambda^{(\kappa)} \frac{\partial \overline{\phi^{(\kappa)}}}{\partial \mathbf{x}_\lambda} + \tilde{\mathbf{c}}_\lambda^{(\kappa)} \frac{\partial \overline{\phi^{(\kappa)}}}{\partial \mathbf{x}_\lambda} \right. \\
& \quad \left. - \left(\frac{\partial \mathbf{v}_\lambda^{(\kappa)}}{\partial t} + \mathbf{v}_\mu^{(\kappa)} \frac{\partial \mathbf{v}_\lambda^{(\kappa)}}{\partial \mathbf{x}_\mu} \right) \frac{\partial \overline{\phi^{(\kappa)}}}{\partial \tilde{\mathbf{c}}_\lambda^{(\kappa)}} - \overline{\phi^{(\kappa)}} \frac{\partial \mathbf{v}_\lambda^{(\kappa)}}{\partial \mathbf{x}_\lambda} - \tilde{\mathbf{c}}_\mu^{(\kappa)} \frac{\partial \overline{\phi^{(\kappa)}}}{\partial \tilde{\mathbf{c}}_\lambda^{(\kappa)}} \frac{\partial \mathbf{v}_\lambda^{(\kappa)}}{\partial \mathbf{x}_\mu} \right\} \\
& = \iiint_{\tilde{\mathbf{c}}^{(\kappa)}} \phi^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) \iiint_{\tilde{\mathbf{c}}^{(i)}} \iint_{|\mathbf{k}|=1} \left\{ f^{(i)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(i)}, t) f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) \right. \\
& \quad \left. - f^{(i)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(i)}, t) f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) \right\} k_C^{(\kappa)} d^2 \mathbf{k} d^3 \tilde{\mathbf{c}}_1^{(i)} d^3 \tilde{\mathbf{c}}^{(\kappa)}
\end{aligned} \right\} \quad (9)
\end{aligned}$$

This is the equation of change of molecular property of the κ -gas of the binary gas mixture. We consider (9) for $\phi^{(\kappa)} = 1$. Putting $\phi^{(\kappa)} = 1$, we can easily have

$$\frac{\partial \mathbf{n}^{(\kappa)}}{\partial t} + \frac{\partial (\mathbf{n}^{(\kappa)} \mathbf{v}_\lambda^{(\kappa)})}{\partial \mathbf{x}_\lambda} = 0 \quad (10)$$

This is equation of continuity, which is the same form for simple gas. For $\phi^{(\kappa)} = m^{(\kappa)} \tilde{\mathbf{c}}_\nu^{(\kappa)}$, we have

$$\begin{aligned}
\overline{\phi^{(\kappa)}} &= 0, \quad \overline{\mathbf{n}^{(\kappa)} \phi^{(\kappa)} \tilde{\mathbf{c}}_\lambda^{(\kappa)}} = -\mathbf{p}_{\lambda\nu}^{(\kappa)}, \quad \frac{\partial \overline{\phi^{(\kappa)}}}{\partial t} = 0, \quad \frac{\partial \overline{\phi^{(\kappa)}}}{\partial \mathbf{x}_\lambda} = 0, \quad \tilde{\mathbf{c}}_\lambda^{(\kappa)} \frac{\partial \overline{\phi^{(\kappa)}}}{\partial \mathbf{x}_\lambda} = 0, \\
\frac{\partial \overline{\phi^{(\kappa)}}}{\partial \tilde{\mathbf{c}}_\lambda^{(\kappa)}} &= m^{(\kappa)} \mathbf{n}^{(\kappa)} \delta_{\lambda\nu}, \quad \tilde{\mathbf{c}}_\lambda^{(\kappa)} \frac{\partial \overline{\phi^{(\kappa)}}}{\partial \tilde{\mathbf{c}}_\lambda^{(\kappa)}} = m^{(\kappa)} \mathbf{n}^{(\kappa)} \tilde{\mathbf{c}}_\lambda^{(\kappa)} \delta_{\lambda\nu} = 0
\end{aligned}$$

and the left side of (9) for $m^{(\kappa)} \tilde{\mathbf{c}}_\nu^{(\kappa)}$ would become

$$\begin{aligned}
& \left. \begin{aligned}
& \frac{\partial \overline{\phi^{(\kappa)}} \mathbf{n}^{(\kappa)}}{\partial t} + \mathbf{v}_\lambda^{(\kappa)} \frac{\partial \overline{\phi^{(\kappa)}} \mathbf{n}^{(\kappa)}}{\partial \mathbf{x}_\lambda} + \frac{\partial}{\partial \mathbf{x}_\lambda} \left(\mathbf{n}^{(\kappa)} \overline{\phi^{(\kappa)}} \tilde{\mathbf{c}}_\lambda^{(\kappa)} \right) - \mathbf{n}^{(\kappa)} \left\{ \frac{\partial \overline{\phi^{(\kappa)}}}{\partial t} + \mathbf{v}_\lambda^{(\kappa)} \frac{\partial \overline{\phi^{(\kappa)}}}{\partial \mathbf{x}_\lambda} + \tilde{\mathbf{c}}_\lambda^{(\kappa)} \frac{\partial \overline{\phi^{(\kappa)}}}{\partial \mathbf{x}_\lambda} \right. \\
& \quad \left. - \left(\frac{\partial \mathbf{v}_\lambda^{(\kappa)}}{\partial t} + \mathbf{v}_\mu^{(\kappa)} \frac{\partial \mathbf{v}_\lambda^{(\kappa)}}{\partial \mathbf{x}_\mu} \right) \frac{\partial \overline{\phi^{(\kappa)}}}{\partial \tilde{\mathbf{c}}_\lambda^{(\kappa)}} - \overline{\phi^{(\kappa)}} \frac{\partial \mathbf{v}_\lambda^{(\kappa)}}{\partial \mathbf{x}_\lambda} - \tilde{\mathbf{c}}_\mu^{(\kappa)} \frac{\partial \overline{\phi^{(\kappa)}}}{\partial \tilde{\mathbf{c}}_\lambda^{(\kappa)}} \frac{\partial \mathbf{v}_\lambda^{(\kappa)}}{\partial \mathbf{x}_\mu} \right\} \\
& = \rho^{(\kappa)} \left(\frac{\partial \mathbf{v}_\nu^{(\kappa)}}{\partial t} + \mathbf{v}_\mu^{(\kappa)} \frac{\partial \mathbf{v}_\nu^{(\kappa)}}{\partial \mathbf{x}_\mu} \right) - \frac{\partial \mathbf{p}_{\nu\mu}^{(\kappa)}}{\partial \mathbf{x}_\mu}, \quad \rho^{(\kappa)} = m^{(\kappa)} \mathbf{n}^{(\kappa)}
\end{aligned} \right\} \quad (11)
\end{aligned}$$

The collision term of the right hand side of (9) for $m^{(\kappa)} \tilde{\mathbf{c}}^{(\kappa)}$ due to collision between particles of the different species for $m^{(\kappa)} \tilde{\mathbf{c}}^{(\kappa)}$ is only considered:

$$\begin{aligned}
& \iiint_{\tilde{\mathbf{c}}^{(\kappa)}} \iiint_{\tilde{\mathbf{c}}^{(i)}} \iint_{|\mathbf{k}|=1} m^{(\kappa)} \tilde{\mathbf{c}}^{(\kappa)} \left\{ f^{(i)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(i)}, t) f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) - f^{(i)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(i)}, t) f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) \right\} k_C^{(\kappa)} (b^{(i\kappa)}, g^{(i\kappa)}) d^2 \mathbf{k} d^3 \tilde{\mathbf{c}}_1^{(i)} d^3 \tilde{\mathbf{c}}^{(\kappa)} \\
& = m^{(\kappa)} \iiint_{\tilde{\mathbf{c}}^{(\kappa)}} \iiint_{\tilde{\mathbf{c}}^{(i)}} \iint_{|\mathbf{k}|=1} \left\{ \tilde{\mathbf{c}}^{(\kappa)} + \frac{2m^{(i)}}{m^{(\kappa)} + m^{(i)}} \left((\tilde{\mathbf{c}}_1^{(i)} - \tilde{\mathbf{c}}^{(\kappa)} + \mathbf{v}^{(i)} - \mathbf{v}^{(\kappa)}) \cdot \mathbf{k} \right) \mathbf{k} \right\} \\
& \quad f^{(i)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(i)}, t) f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) k_C^{(\kappa)} (b^{(i\kappa)}, g^{(i\kappa)}) d^2 \mathbf{k} J^{(i)} d^3 \tilde{\mathbf{c}}_1^{(i)} J^{(\kappa)} d^3 \tilde{\mathbf{c}}^{(\kappa)} \\
& - m^{(\kappa)} \iiint_{\tilde{\mathbf{c}}^{(\kappa)}} \iiint_{\tilde{\mathbf{c}}^{(i)}} \iint_{|\mathbf{k}|=1} \tilde{\mathbf{c}}^{(\kappa)} f^{(i)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(i)}, t) f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) k_C^{(\kappa)} (b^{(i\kappa)}, g^{(i\kappa)}) d^2 \mathbf{k} d^3 \tilde{\mathbf{c}}_1^{(i)} d^3 \tilde{\mathbf{c}}^{(\kappa)}
\end{aligned}$$

where $J^{(i)}(\mathbf{k}) = \partial(\tilde{\mathbf{c}}_1^{(i)}) / \partial(\tilde{\mathbf{c}}_1^{(i)})$ and $J^{(\kappa)}(\mathbf{k}) = \partial(\tilde{\mathbf{c}}^{(\kappa)}) / \partial(\tilde{\mathbf{c}}^{(\kappa)})$ are Jacobians. Collision dynamics (4) have been used. Thus, we have approximately as the right hand side of (9) for $m^{(\kappa)} \tilde{\mathbf{c}}^{(\kappa)}$ due to $f^{(i)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(i)}, t) \approx f_M^{(i)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(i)}, t)$,

$f^{(l)}(\mathbf{x}, \tilde{\mathbf{c}}^{(l)}, t) \approx f_M^{(l)}(\mathbf{x}, \tilde{\mathbf{c}}^{(l)}, t)$, $f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) \approx f_M^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t)$ and $f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) \approx f_M^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t)$,

$$\begin{aligned}
& m^{(\kappa)} \iiint_{\tilde{\mathbf{c}}^{(\kappa)}} \iiint_{\tilde{\mathbf{c}}^{(l)}} \iint_{|\mathbf{k}|=1} \left\{ \tilde{\mathbf{c}}^{(\kappa)} + \frac{2m^{(l)}}{m^{(\kappa)} + m^{(l)}} \left((\tilde{\mathbf{c}}_1^{(l)} - \tilde{\mathbf{c}}^{(\kappa)} + \mathbf{v}^{(l)} - \mathbf{v}^{(\kappa)}) \cdot \mathbf{k} \right) \mathbf{k} \right\} \\
& f^{(l)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(l)}, t) f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) k_C^{(\kappa)}(b^{(\kappa)}, g^{(\kappa)}) d^2 \mathbf{k} J^{(l)} d^3 \tilde{\mathbf{c}}_1^{(l)} J^{(\kappa)} d^3 \tilde{\mathbf{c}}^{(\kappa)} \\
& - m^{(\kappa)} \iiint_{\tilde{\mathbf{c}}^{(\kappa)}} \iiint_{\tilde{\mathbf{c}}^{(l)}} \iint_{|\mathbf{k}|=1} \tilde{\mathbf{c}}^{(\kappa)} f^{(l)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(l)}, t) f^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) k_C^{(\kappa)}(b^{(\kappa)}, g^{(\kappa)}) d^2 \mathbf{k} d^3 \tilde{\mathbf{c}}_1^{(l)} d^3 \tilde{\mathbf{c}}^{(\kappa)} \\
& \approx \frac{2m^{(l)}m^{(\kappa)}}{m^{(l)} + m^{(\kappa)}} (\mathbf{v}_\mu^{(l)} - \mathbf{v}_\mu^{(\kappa)}) \iiint_{\tilde{\mathbf{c}}^{(\kappa)}} \iiint_{\tilde{\mathbf{c}}^{(l)}} \iint_{|\mathbf{k}|=1} (\mathbf{e}^{(\kappa)} \otimes \mathbf{k}^T)_{\lambda\mu} \\
& f_M^{(l)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(l)}, t) f_M^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) J^{(l)}(\mathbf{k}) J^{(\kappa)}(\mathbf{k}) k_C^{(\kappa)}(b^{(\kappa)}, g^{(\kappa)}) d^2 \mathbf{k} d^3 \tilde{\mathbf{c}}_1^{(l)} d^3 \tilde{\mathbf{c}}^{(\kappa)} \\
& = \eta_{\lambda\mu}^{(\kappa)} (\mathbf{v}_\mu^{(l)} - \mathbf{v}_\mu^{(\kappa)}) \equiv F_{\lambda}^{(\kappa)} \quad , \quad g^{(\kappa)} = |\mathbf{c}_1^{(l)} - \mathbf{c}^{(\kappa)}|
\end{aligned} \tag{12}$$

where $\mathbf{e}^{(\kappa)} = (\mathbf{v}^{(l)} - \mathbf{v}^{(\kappa)}) / |\mathbf{v}^{(l)} - \mathbf{v}^{(\kappa)}|$ and $F_{\lambda}^{(\kappa)}$ is a body force from l -gas to κ -gas and

$$(\mathbf{e}^{(\kappa)} \otimes \mathbf{k}^T)_{\lambda\mu} = \begin{pmatrix} e_1^{(\kappa)} \\ e_2^{(\kappa)} \\ e_3^{(\kappa)} \end{pmatrix} \otimes (k_1 \ k_2 \ k_3) = \begin{pmatrix} e_1^{(\kappa)} k_1 & e_1^{(\kappa)} k_2 & e_1^{(\kappa)} k_3 \\ e_2^{(\kappa)} k_1 & e_2^{(\kappa)} k_2 & e_2^{(\kappa)} k_3 \\ e_3^{(\kappa)} k_1 & e_3^{(\kappa)} k_2 & e_3^{(\kappa)} k_3 \end{pmatrix}$$

Thus we have

$$\left. \begin{aligned}
\eta_{\lambda\mu}^{(\kappa)} &= \frac{2m^{(l)}m^{(\kappa)}}{m^{(l)} + m^{(\kappa)}} \iiint_{\tilde{\mathbf{c}}^{(\kappa)}} \iiint_{\tilde{\mathbf{c}}^{(l)}} \iint_{|\mathbf{k}|=1} (\mathbf{e}^{(\kappa)} \otimes \mathbf{k}^T)_{\lambda\mu} \\
& f_M^{(l)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(l)}, t) f_M^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}^{(\kappa)}, t) J^{(l)}(\mathbf{k}) J^{(\kappa)}(\mathbf{k}) k_C^{(\kappa)}(b^{(\kappa)}, g^{(\kappa)}) d^2 \mathbf{k} d^3 \tilde{\mathbf{c}}_1^{(l)} d^3 \tilde{\mathbf{c}}^{(\kappa)} \\
& = \frac{2m^{(l)}m^{(\kappa)}}{m^{(l)} + m^{(\kappa)}} \int (\mathbf{e}^{(\kappa)} \otimes \mathbf{k}^T)_{\lambda\mu} k_C^{(\kappa)}(b^{(\kappa)}, g^{(\kappa)}) d\chi^{(\kappa)}
\end{aligned} \right\} \tag{13}$$

Combining (11) and (13), we have the equations of motion for κ -gas:

$$\left. \begin{aligned}
\rho^{(\kappa)} \left(\frac{\partial \mathbf{v}_v^{(\kappa)}}{\partial t} + \mathbf{v}_\mu^{(\kappa)} \frac{\partial \mathbf{v}_v^{(\kappa)}}{\partial \mathbf{x}_\mu} \right) &= \frac{\partial \mathbf{p}_{v\mu}^{(\kappa)}}{\partial \mathbf{x}_\mu} + \eta_{v\mu}^{(\kappa)} (\mathbf{v}_\mu^{(l)} - \mathbf{v}_\mu^{(\kappa)}) \\
p_{v\lambda}^{(\kappa)} &= -p^{(\kappa)} \delta_{v\lambda} + \mu^{(\kappa)} \left(\frac{\partial \mathbf{v}_\lambda^{(\kappa)}}{\partial \mathbf{x}_v} + \frac{\partial \mathbf{v}_v^{(\kappa)}}{\partial \mathbf{x}_\lambda} \right),
\end{aligned} \right\} \quad (\lambda, \mu, v): \text{cyclic} \tag{14}$$

These are the equations of motion for κ -gas. The last term of the first equation of (14) is a new term, showing the force acting from l to κ and vice versa.

EQUATIONS OF ENERGY

We are going to get the energy equations on the basis of (9). Taking $\phi^{(\kappa)} = (1/2)m^{(\kappa)}(\tilde{\mathbf{c}}^{(\kappa)})^2$, we have

$$\begin{aligned}
\overline{\phi^{(\kappa)}} &= \frac{3}{2} k_B T^{(\kappa)}, \quad n^{(\kappa)} \overline{\phi^{(\kappa)} \tilde{c}_\lambda^{(\kappa)}} = \frac{3}{2} m^{(\kappa)} \overline{\tilde{\mathbf{c}}^{(\kappa)2} \tilde{c}_\lambda^{(\kappa)}}, \quad \frac{\partial \overline{\phi^{(\kappa)}}}{\partial t} = 0, \quad \frac{\partial \overline{\phi^{(\kappa)}}}{\partial \mathbf{x}_\lambda} = 0, \\
\overline{\tilde{c}_\lambda^{(\kappa)} \frac{\partial \phi^{(\kappa)}}{\partial \mathbf{x}_\lambda}} &= 0, \quad \frac{\partial \overline{\phi^{(\kappa)}}}{\partial \tilde{c}_\lambda^{(\kappa)}} = 0, \quad n^{(\kappa)} \tilde{c}_\lambda^{(\kappa)} \frac{\partial \overline{\phi^{(\kappa)}}}{\partial \tilde{c}_\mu^{(\kappa)}} = m^{(\kappa)} n^{(\kappa)} \overline{\tilde{c}_\lambda^{(\kappa)} \tilde{c}_\mu^{(\kappa)}} = -p_{\lambda\mu}^{(\kappa)}
\end{aligned}$$

Thus, we have

$$\left. \begin{aligned} & \frac{3}{2} k_B n^{(\kappa)} \left(\frac{\partial T^{(\kappa)}}{\partial t} + \mathbf{v}_\mu^{(\kappa)} \frac{\partial T^{(\kappa)}}{\partial x_\mu} \right) + \frac{3}{2} \frac{\partial}{\partial x_\lambda} \left(m^{(\kappa)} \overline{\tilde{\mathbf{c}}^{(\kappa)2} \tilde{c}_\lambda^{(\kappa)}} \right) - p_{\lambda\mu}^{(\kappa)} \frac{\partial v_\lambda^{(l)}}{\partial x_\mu} \\ & = -K_T^{(\kappa)} (T^{(\kappa)} - T^{(l)}) + K_V^{(\kappa)} (\mathbf{v}^{(l)} - \mathbf{v}^{(\kappa)})^2 \end{aligned} \right\} \quad (15)$$

The time scale of $\partial T^{(\kappa)}/\partial t$ corresponds to that of macroscopic motion τ_{mac} . On the other hand, the translational energy of a molecule $(3/2)m^{(\kappa)}(\mathbf{c}^{(\kappa)})^2$ would be in equilibrium of internal energy of the molecule, so that the time scale of the thermal equilibrium would be a couple of the mean free time τ_{mf} between collision. It could be assumed $\tau_{mac} \gg \tau_{mf}$. According the above discussions, the first term could be written as $(1/2)k_B n^{(\kappa)} N^{(\kappa)} (\partial T^{(\kappa)}/\partial t + \mathbf{v}_\mu^{(\kappa)} \partial T^{(\kappa)}/\partial x_\mu)$ and the second term would be written as $\partial q_\lambda^{(\kappa)}/\partial x_\lambda$, *i.e.*

$$\left. \begin{aligned} & \frac{1}{2} N^{(\kappa)} k_B n^{(\kappa)} \left(\frac{\partial T^{(\kappa)}}{\partial t} + \mathbf{v}_\mu^{(\kappa)} \frac{\partial T^{(\kappa)}}{\partial x_\mu} \right) + \frac{\partial q_\lambda^{(\kappa)}}{\partial x_\lambda} - p_{\lambda\mu}^{(\kappa)} \frac{\partial v_\lambda^{(l)}}{\partial x_\mu} \\ & = -K_T^{(\kappa)} (T^{(\kappa)} - T^{(l)}) + K_V^{(\kappa)} (\mathbf{v}^{(l)} - \mathbf{v}^{(\kappa)})^2 \\ & \quad q_\lambda^{(\kappa)} = \frac{1}{2} N^{(\kappa)} m^{(\kappa)} \overline{(\tilde{\mathbf{c}}^{(\kappa)})^2 \tilde{c}_\lambda^{(\kappa)}} \end{aligned} \right\} \quad (16)$$

where $N^{(\kappa)}$ is the total degrees of freedom of κ -molecule so that for monatomic molecules $N^{(\kappa)} = 3$, $K_T^{(\kappa)}$ and $K_V^{(\kappa)}$ are also approximately written as

$$\left. \begin{aligned} K_T^{(\kappa)} &= \frac{6m^{(l)}m^{(\kappa)}}{(m^{(l)} + m^{(\kappa)})^2} k_B \iiint_{\tilde{\mathbf{c}}^{(\kappa)}} \iiint_{\tilde{\mathbf{c}}^{(l)}} \iint_{|\mathbf{k}|=1} (\mathbf{e}^{(l\kappa)} \cdot \mathbf{k}) \\ & \quad f_M^{(l)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(l)}, t) f_M^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(\kappa)}, t) J^{(l)}(\mathbf{k}) J^{(\kappa)}(\mathbf{k}) k_C^{(l\kappa)}(b^{(l\kappa)}, g^{(l\kappa)}) d^2 \mathbf{k} d^3 \tilde{\mathbf{c}}_1^{(l)} d^3 \tilde{\mathbf{c}}_1^{(\kappa)} \\ &= \frac{6m^{(l)}m^{(\kappa)}}{(m^{(l)} + m^{(\kappa)})^2} k_B n^{(l)} n^{(\kappa)} \int (\mathbf{e}^{(l\kappa)} \cdot \mathbf{k}) k_C^{(l\kappa)}(b^{(l\kappa)}, g^{(l\kappa)}) d\chi^{(l\kappa)} \\ K_V^{(\kappa)} &= \frac{2m^{(l)2}m^{(\kappa)}}{(m^{(l)} + m^{(\kappa)})^2} \iiint_{\tilde{\mathbf{c}}^{(\kappa)}} \iiint_{\tilde{\mathbf{c}}^{(l)}} \iint_{|\mathbf{k}|=1} (\mathbf{e}^{(l\kappa)} \cdot \mathbf{k}) \\ & \quad f_M^{(l)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(l)}, t) f_M^{(\kappa)}(\mathbf{x}, \tilde{\mathbf{c}}_1^{(\kappa)}, t) J^{(l)}(\mathbf{k}) J^{(\kappa)}(\mathbf{k}) k_C^{(l\kappa)}(b^{(l\kappa)}, g^{(l\kappa)}) d^2 \mathbf{k} d^3 \tilde{\mathbf{c}}_1^{(l)} d^3 \tilde{\mathbf{c}}_1^{(\kappa)} \\ &= \frac{2m^{(l)2}m^{(\kappa)}}{(m^{(l)} + m^{(\kappa)})^2} n^{(l)} n^{(\kappa)} \int (\mathbf{e}^{(l\kappa)} \cdot \mathbf{k}) k_C^{(l\kappa)}(b^{(l\kappa)}, g^{(l\kappa)}) d\chi^{(l\kappa)} \\ d^2 \mathbf{k} &= d\chi^{(l\kappa)} d\phi^{(l\kappa)} \end{aligned} \right\} \quad (17)$$

where the term $K_T^{(\kappa)}(T^{(\kappa)} - T^{(l)})$ and $K_V^{(\kappa)}(\mathbf{v}^{(l)} - \mathbf{v}^{(\kappa)})^2$ designate the relaxations of temperature and velocity, and $\phi^{(l\kappa)}$ is a polar angle giving the orientation of $\mathbf{e}^{(l\kappa)}$ about an axis parallel to $\mathbf{e}^{(l\kappa)}$ which takes all values from 0 to 2π .

CONCLUSIONS

We have had the hydrodynamic equations for binary gas mixture from Boltzmann equations:

$$\frac{\partial \mathbf{n}^{(\kappa)}}{\partial t} + \frac{\partial (\mathbf{n}^{(\kappa)} \mathbf{v}_\lambda^{(\kappa)})}{\partial \mathbf{x}_\lambda} = 0 \quad (18)$$

$$\left. \begin{aligned} \rho^{(\kappa)} \left(\frac{\partial \mathbf{v}_\nu^{(\kappa)}}{\partial t} + \mathbf{v}_\mu^{(\kappa)} \frac{\partial \mathbf{v}_\nu^{(\kappa)}}{\partial \mathbf{x}_\mu} \right) &= \frac{\partial \mathbf{p}_{\nu\mu}^{(\kappa)}}{\partial \mathbf{x}_\mu} + \eta_{\nu\mu}^{(\kappa)} (\mathbf{v}_\mu^{(\iota)} - \mathbf{v}_\mu^{(\kappa)}) \\ \mathbf{p}_{\nu\lambda}^{(\kappa)} &= -p \delta_{\nu\lambda} + \mu^{(\kappa)} \left(\frac{\partial \mathbf{v}_\lambda^{(\kappa)}}{\partial \mathbf{x}_\nu} + \frac{\partial \mathbf{v}_\nu^{(\kappa)}}{\partial \mathbf{x}_\lambda} \right), \end{aligned} \right\} (\lambda, \mu, \nu): \text{cyclic} \quad (19)$$

$$\left. \begin{aligned} \frac{1}{2} k_B n^{(\kappa)} N^{(\kappa)} \left(\frac{\partial T^{(\kappa)}}{\partial t} + \mathbf{v}_\mu^{(\kappa)} \frac{\partial T^{(\kappa)}}{\partial \mathbf{x}_\mu} \right) + \frac{\partial \mathbf{q}_\lambda^{(\kappa)}}{\partial \mathbf{x}_\lambda} + \mathbf{p}_{\lambda\mu}^{(\kappa)} \frac{\partial \mathbf{v}_\lambda^{(\iota)}}{\partial \mathbf{x}_\mu} \\ = -K_T^{(\kappa)} (T^{(\kappa)} - T^{(\iota)}) + K_V^{(\kappa)} (\mathbf{v}^{(\iota)} - \mathbf{v}^{(\kappa)})^2 \end{aligned} \right\} \quad (20)$$

where we have new term showing the relaxation due to differences of velocities between both gases in equations of motion, by comparing equations for simple gas. Further, new two terms due to the relaxations of velocities and temperatures are obtained in equations of energy. $\eta_{\lambda\mu}^{(\kappa)}$, $K_T^{(\kappa)}$ and $K_V^{(\kappa)}$ are shown in (13) and (16). These equations are similar to the former ones of S.K. in the form.

SUMMARY

Hydrodynamic equations for binary gas mixture are derived from Boltzmann equations. Equations of continuity for each gas are of the same form for simple gas. Equations of motion have additional terms of velocity relaxation in comparison with simple gas, which are linear in difference of velocities of each gas. Energy equations have two additional terms, one of which is due to velocity relaxation between two gases. The other is that due to temperature relaxation between those.

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